Education, Rent-seeking and the Curse of Natural Resources.

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Abstract

Empirical evidence suggests that natural resources breed corruption and reduce educational attainments, dampening economic growth. The theoretical literature has treated these two channels separately, with natural resources affecting growth either through human capital or corruption. In this paper, we argue that education and corruption are jointly determined and depend on the endowment of natural resources. Natural resources affect the incentives to invest in education and rent seeking that in turn affect growth. Whether natural resources stimulate growth or induce a poverty-trap crucially depends on inequality in access to education and political participation, as well as on the cost of political participation. For lower inequality and higher cost of political participation, a high-growth and a poverty-trap equilibrium co-exist even with abundant natural resources.

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1 Introduction

Recent evidence shows that the abundance of natural resources can be a curse as it may dampen the accumulation of other productive capitals such as human capital and social capital (here specifically, absence of corruption). Gylfason (2001) finds that across countries, school enrollment at all levels is inversely related to the abundance of natural resources. Another strand of literature shows that countries rich in natural resources tend to generate higher rents and seem more prone to rent-seeking. Rent seeking in turn prevents growth and development. Ross (2001) reports that in Southeast Asia (Philippines, Indonesia and Malaysia) a hardwood timber price boom created rents that compelled political elites to alter the institutions to acquire greater control over resource rents, and as a result political power became more concentrated and corruption increased. Furthermore, social capital and human capital also reinforce each other. Empirical evidence suggests that corruption and education are inversely correlated (Svensson 2005, Glaser et al. 2004). Yet, the theoretical literature has treated them separately with natural resources affecting growth either through human capital (Gylfason et al. 1999, Torvik 2001, Bravo-Ortega and Gregorio 2005) or through rent seeking (Baland and Francois 2000, Torvik 2002). Others show a joint determination of corruption and education (Ehrlich and Lui 1999, and Eicher, García-Peñalosa and van Ypersele 2009). In this paper we combine all three strands of literature and argue that education and rent seeking are jointly determined. Natural resources affect both human capital and corruption, implying the co-movements among these three variables observed in empirical evidence.

The last few years have witnessed a growing evidence that countries rich in natural resources perform worse than those without resources (Sach and Warner 1995, 1999, 2001, Gylfason et al. 1999, Gylfason 2001, Auty 2001, Birdsall et al. 2001, Robinson et al. 2006, Mehlum et al. 2006). On average natural resource rich countries have slower growth, lower education, bad institutions and more corruption. Gylfason (2001) reports that for OPEC countries, GNP per capita decreased by 1.3 percent per year on average during 1965-98 compared with 2.2 percent average per capita growth in all lower- and middle-income countries. Nigeria provides a perfect example of resource curse. Oil revenues per capita in Nigeria increased from US$33 in 1965 to US$325 in 2000, but income per capita has stagnated at around US$1100 in PPP terms since its independence in 1960 putting Nigeria among the 15 poorest countries in the world (Bevan et al. 1999).

On the other hand, the success stories of some resource abundant countries, i.e. USA, Canada, Norway, and Botswana highlight the fact that resource abundance is not always associated with poor economic performance. In these cases, the resource boom stimulated growth and improved welfare.[1] Prime example

[1] Stijns (2006) even contests the claim that resource abundance crowds out human capital.

that can be cited is the Botswana. Forty percent of Botswana's GDP stems from diamonds, but Botswana has managed to beat the resource curse. It has the second highest public expenditure on education as a fraction of GNP, it enjoys the world's highest growth rate since 1965 and its GDP per capita is at least ten times that of Nigeria (Sarraf and Jiwanji, 2001). In their empirical investigation, Mehlum, Moene and Torvik (2006) find that whether resources are a curse or a blessing crucially depends on whether the institutions are grabber-friendly or producer-friendly. They conclude that institutional quality is the key to understanding the resource curse: when institutions are bad, resource abundance is a curse; when institutions are good, resource abundance is a blessing. Their findings coincide with those of Gylfason (2001) who finds that the relationship between school enrollment and natural resource abundance is non-monotonic. Thus, empirically, higher resource abundance can be compatible with higher growth, higher investment in human capital and lower corruption.

Theoretically, the 'resource curse' has mainly been explained by two types of theories; market-based and political economy based. Market-based explanations rely on the 'Dutch disease' hypothesis, where a resource boom is linked to a crowding out of manufacturing exports. Lately, focus has shifted toward other crowding out effects especially on the forgone investment in human capital. According to this view, the natural resource sector is considered to be unskilled and it does not generate learning by doing and spillover effects. The abundance of natural resources shifts factors of production away from the manufacturing sector that generates learning by doing, thus, reducing productivity growth (Gylfason et al. 1999, Bravo-Ortega and Gregorio 2005). This is further extended in Torvik (2001), where all sectors contribute to learning by doing and there are spillover among them. He concludes that whether abundance of natural resources reduces the growth or not depends on the structural characteristics of the economy.

On the other hand, the political economy argument maintains that the presence of natural resources creates rents, especially when institutions are weak it may give rise to "voracity effects". Interest groups divert their time and energies to capture these rents, which results in a miss-allocation of time and talent. Torvik (2002) argues that natural resource abundance increases the number of entrepreneurs engaged in rent seeking and reduces the number of entrepreneurs running productive firms. More natural resources, thus, lead to a lower welfare. Baland and Francois (2000) provide conditions under which resource booms lead to an increase in rent-seeking activity. Lane and Tornell (1999) and Auty (2001) also show that the resource curse operates through rent-seeking and voracity effects.

Our contribution in this paper is twofold. First, in contrast to the existing theoretical literature which generally considers low educational attainment, corruption and abundant natural resources separately, or at best in pairs, we combine all three strands of literature where natural resources affect both education and corruption. We argue that human capital and rent seeking are jointly determined, where in our case human capital implies formal schooling rather than learning by doing.[2] Second, in line with the diversified experiences of resource rich countries, we show that the relationship between 'resource abundance' and 'resource curse' is non-monotonic. Our model assumptions imply that human capital is the only productive capital. Our treatment of human capital differs from those of Gylfason et al. (1999) and Bravo-Ortega and Gregorio (2005) who use learning by doing mechanisms. First, both of these models are based on the Dutch disease hypothesis that has failed to fetch empirical support. Sala-i-Martin and Subramanian (2003) find that corruption, and granting of import licenses rather than Dutch disease are the reasons why oil richness of Nigeria turned into a curse.[3] Second, we use a concept of formal education that captures more closely the measures used in empirical literature as compared to learning-by-doing. Third, they do not consider rent seeking neither do they allow for heterogeneous agents.

We develop an endogenous growth model with two sectors; an industrial sector and a natural resource sector. The industrial sector employs human capital, whereas the natural resource sector uses natural resources and unskilled labor.[4] Although the natural resource sector does not employ human capital, there is a positive externality from human capital accumulation to production in the natural resource sector. The central element in our analysis is the decision to accumulate political capital that enables agents to divert rents from the natural resource sector, and the trade-off between investing in human and political capitals. Individuals can divide their time between working and investing in human and/or political capital accumulation. Our setting has common features with Ehrlich and Lui (1999) who look at the trade-off between corruption and human capital but do not consider natural resources. Our model differs from theirs in a number of ways: the introduction of a natural resource sector, the source of heterogeneity among the agents and the monetary costs associated with corruption (quality of institutions).

Natural resources are generally state owned and if institutions are weak, they can easily be appropriated by the ruling class. Thus it provides them with incentives to invest in accumulating political power and extracting the rents accruing from natural resources. We assume that access to political participation depends on the class one belongs to. We consider a source of heterogeneity that captures the patterns often observed in developing countries where the right to rule and access to education is confined to specific

[2] This is important as the empirical studies use the data on education that represent the formal schooling rather than learning by doing.

[3] See Bulte et al. (2005) for more discussion on the empirical failure of Dutch disease hypothesis.

[4] In our settings, there is no skilled labor competition between industrial and natural resource sectors, a mechanism that is used in the Dutch disease literature. Since, we consider formal schooling rather than learning by doing (in Dutch disease literature, only industrial sector generates learning by doing), labor movements do not play very significant role.

ethnic or linguistic groups. We suppose that society is divided into two groups; the elite and the non-elite (workers). The elite have access to the technologies for political and human capital accumulation, while the workers do not.

Another new element in our model is that we emphasize the fact that institutions play a pivotal role for the returns of rent seeking. We introduce a monetary cost associated with rent seeking that depicts the quality of institutions. The better quality institutions would imply more costly rent seeking. Thus, lower the cost is, the more conducive would be the institutions for rent seeking and the more profitable would be the investment in political capital. In this way, although, natural resources generate rents that always invite rent seeking, given the cost of political participation, it might not always be profitable to be corrupt.

We find that the economy may exhibit a high-growth equilibrium, with fast growth, high accumulation of human capital and no corruption (no rent seeking); a low-growth equilibrium, with slow growth, low investment in both human and political capitals; and a poverty-trap equilibrium, with no growth, no investment in human capital, and higher corruption. In which of these three equilibria the economy is depends on the endowment of natural resources. There are endogenous thresholds of natural resources that demarcate different equilibria. For low abundance of natural resources, there is a unique high-growth equilibrium and for high abundance, there is a unique poverty-trap equilibrium; while for intermediate levels of natural resources, there are multiple equilibria where the high-growth equilibrium either coexists with the poverty-trap equilibrium or with the low-growth equilibrium. The thresholds that demarcate different equilibria are endogenous and crucially depend on two parameters; inequality in access to education and political participation, and the monetary cost of political participation (quality of institutions). Depending on these two parameters, the poverty-trap and the high-growth equilibrium can coexist. Particularly, increasing access to education and political participation would increase the range of natural resource endowments where there is a high-growth equilibrium and would decrease the range of natural resources where there is a poverty-trap equilibrium. Similarly, increasing the quality of institutions would reduce the returns to rent seeking and would increase the range of natural resource endowments where there is a high-growth equilibrium.

The rest of paper proceeds as follows. In the next section we define our model. Section 3 solves the model, while section 4 characterizes the different equilibria and examines the comparative statics. The last section concludes.

2 Model assumptions

2.1 Households

There is an overlapping generations economy of population N of two-period lived individuals. Agents within each generation are differentiated by the access to human and political capital accumulation technologies. Of those, n are the 'elite' who have access to both human and political capital accumulation technologies. Others, $l = N - n$ are unskilled workers, they do not have access to the technologies for accumulating human or political capital, and work in natural resource sector.[5] In this way the source of heterogeneity is inequality in access to education and political participation, while within each group agents are homogeneous.[6] All agents are endowed with one divisible unit of time in each period. The elite divide their time between schooling, accumulating political capital and working in the manufacturing sector while unskilled workers spend their entire time working in the natural resource sector.

All individuals born at $t = 0, 1, 2, ...$ have identical preferences, represented by the utility function

$$U = c_{1t} + \beta c_{2t} \tag{1}$$

where $\beta < 1$ is the discount factor and c_{1t} and c_{2t} is the consumption in first and second period, respectively. Linear utility function allows us to focus on the income effects that are more important for our problem than the intertemporal substitution effects.

2.2 Technologies

The accumulation of human capital is the engine of growth. Only the elite have access to education and political process. They invest in human capital when young and in political capital when old. In this way, returns to education would depend on the investment in political capital and it will augment a trade-off between two capitals that we are interested in. They are endowed with one unit of time in each period. In the first period they divide this time between accumulating human capital H_t and working in the manufacturing sector. Let h_{it} be the time spent by the ith member of the elite in accumulating human capital so that $1 - h_{it}$ is the time spent for working. In the second period he spends time q_{it} in accumulating political capital so that $1 - q_{it}$ is the time spent for working. The stock of human (political) capital of individual when young is the inherited human (political) capital from his parents. In the second

[5] This is one type of the heterogeneity that we incorporate, their can, of course be another source of heterogeneity where may be both groups have access to education while only one group has access to political process.

[6] This is, of course, a simplification which allows us to solve the model smoothly, other specification can be when the elite are also heterogeneous.

period, his human (political) capital is a function of his inherited stock of human (political) capital and the time devoted to its accumulation when young.

$$H_{i2t} = Ah_{it}^{\theta} H_{i1t} \qquad\qquad 0 < \theta < 1 \tag{2}$$

where H_{i1t} denotes individual's inherited stock of human capital, $H_{i1t} = H_{i2t-1}$, h_{it} the fraction of time invested in creating human capital and A a technological parameter.

$$Q_{i2t} = Bq_{it}^{\gamma} Q_{i1t} \qquad\qquad 0 < \gamma < 1 \tag{3}$$

where Q_{i1t} denotes individual's inherited stock of political capital, $Q_{i1t} = Q_{i2t-1}$, q_{it} the fraction of time invested in creating political capital and B a technological parameter.

There are two sectors in the economy: the first one produces a manufacturing good and second one consists of extraction of the natural resources. The manufacturing sector employs human capital, with every skilled worker producing one unit of output. Aggregate output in manufacturing sector is

$$Y_t^m = \sum_i \left[(1 - h_{it})H_{i1t} + (1 - q_{it-1})H_{i2t-1}\right] \tag{4}$$

Individual i produces $Y_{i1t}^m = (1 - h_{it})H_{i1t}$ when young and $Y_{i2t}^m = (1 - q_{it})H_{i2t}$ when old. Equation (4) implies that the source of growth is the accumulation of human capital.

Output in the natural resource sector depends on the stock of natural resources R, which is given and constant.[7] Output is produced with a constant returns technology of the form

$$Y_t^R = a_t R^{\alpha} l^{1-\alpha} \qquad\qquad 0 < \alpha < 1 \tag{5}$$

where a_t is the level of technology and l is unskilled labor. As in Bravo-Ortega and Gregorio (2005), R represents a measure of the endowment of the natural resources and its impact on output. This sector benefits from an externality arising from the human capital accumulated by the skilled workers. In particular, we assume that the externality depends on the average stock of human capital of the current generation, $a_t = \max\{\underline{a}, aH_{2t}\}$, with a being positive constant and H_{2t} is average stock of human capital.[8]

[7] This assumption is similar to those used by Bravo-Ortega and Gregorio (2005), Aldave Ruiz and García-Peñalosa (2009) and Matsuyama (1992). Allowing for depletion of the natural resources would tend to imply decreasing R over time, hence, output and the rents. We refrain from the normative analysis of the natural resource management of those Stiglitz (1974), Solow (1974), Dasgupta and Heal (1974), Mitra (2002), d'Albis and Ambec (2010). Of course, natural resource management is one of the most important aspects of resource economics, but here, we focus on rents appropriation.

[8] The assumption that production technology in the natural resource sector increases with average stock of human capital is not crucial for our results, we need it to ensure that the natural resource sector (so the rents) is not disappeared in the long run.

In this way, although, the natural resources sector does not employ skilled labor but its production grows with the human capital. Our formulation of a_t implies that when there is no human capital accumulation, the productivity in the natural resource sector will be at its lower bound, $a_t = \underline{a}$.

2.3 Rents and political behavior

Both sectors are competitive; in the manufacturing sector workers are paid their marginal product and output is exhausted by the payments of wages. In the natural resources sector, labor is paid their marginal product thus the wage bill is equal to $(1-\alpha)Y_t^R$. Natural resources are state owned and the remaining output αY_t^R are the rents accrued to the government. The elite who have access to political technology may become corrupt and may exploit their offices to divert these rents toward their pockets. Corruption allows a member of the elite to divert a fraction d_{it} of rents to his own pocket. Since all members of the elite are identical, they divert d_t share of rents. The remaining share $(1-d_t)\alpha Y_t^R$ is held by the government who then distributes it equally among all old agents.[9] In this way every old agent whether elite or worker receives a transfer $\frac{(1-d_t)\alpha Y_t^R}{N}$ from the government. The idea here is that members of the elite get a higher share of rents from the natural resource sector when they are corrupt than when they are not.

Corrupt agents exert two externalities on each other. First, there is a positive externality; a higher number of corrupt agents make it easier to divert rents from the natural resource sector. In particular, we assume that the fraction of rents diverted is a function of the proportion of the corrupt elite, that is $d_t\ (p_t) = p_t$. With this functional form, the diverted share is an increasing function of the number of corrupt individuals, with $d(0) = 0$ and $d(1) = 1$. This implies that in the absence of corruption, the elite get no rents and if they are all corrupt they divert all rents and share among themselves. This functional form ensures that if only one agent is corrupt, he can only divert a share equal to $\frac{1}{n}$ of available rents.

There is also a negative externality arising from the way these rents are shared amongst corrupt agents, as in Ehrilich and Lui (1999). What share an individual gets depends on the size of his political capital relative to aggregate political capital. The idea is that one's ability to collect rents depends on how large one's personal power is relative to that of the others. In this way the share that he gets not only depends on his own political capital but also on the political capital of others. The share of rents

[9] Transfers are introduced in the second period to have symmetry between corruption and non-corrupt equilibria. Since, agents extract rents (a share d_t) in the second period when they invest in political capital, thus, the remaining share $(1-d_t)$ is transferred in the same period.

that individual i gets, s_{it}, is simply equal to his share in aggregate political capital, that is

$$s_{it} = \frac{Q_{it}}{nQ_t} = \frac{Q_{it}}{Q_{it} + (n-1)Q_t^-} \tag{6}$$

where Q_{it} is individual i's political capital, nQ_t is the aggregate political capital. We suppose that the size of the elite, n is small such that every individual takes into account the fact that his investment in political capital is going to increase aggregate stock of political capital. Thus, aggregate political capital nQ_t can be decomposed into the political capital of individual i, Q_{it} and the sum of political capital of all others, $(n-1)Q_t^-$. Since all elite are identical, in equilibrium they will have the same stock of political capital, $Q_{it} = Q_t$, and each of them will obtain a fraction $\frac{1}{n}$ of rents; however, each choosing his investments takes Q_t^- as given.[10] This type of rent sharing technology may not be suitable for the cases where members of the elite collude to extract higher share of rents. Of course, there can be cases where elite collude to extract higher share of rents. Introduction of collusive rent sharing would require a different setting that allow for repeated interactions among the elite.

We further suppose that rent seeking is associated with some monetary cost. It can be the direct monetary cost of participation in rent seeking (i.e. political participation) or the loss of revenues to remain inconspicuous. It can also be viewed as the loss/fine incurred if the agent is caught being corrupt and is punished. Given this, it is plausible to assume this monetary cost z to be proportional to their second period income, where $z \in (0,1)$. This implies that corrupt agents not only lose part of their illegal income but they also lose part of their legal income (i.e. if there are fines). This cost can be linked to the quality of institutions; with better quality institutions rent seeking will be more costly than with low quality institutions.

Given this, we can then express the life time income of a corrupt individual as

$$I_{ict} = (1-h_{it})H_{i1t} + (1-z)\left[(1-q_{it})H_{i2t} + \alpha Y_{2t}^R \frac{1-d_t}{N} + \alpha Y_{2t}^R d_t s_{it}\right] \tag{7}$$

In the first period the elite get only wage income. In the second period, the first term is wage income, the second term is transfers, and the third term is rents captured through corruption. The magnitude of corruption depends on both d_t i.e. the share of rents diverted and s_{it} i.e. the share of individual out of it. Since, what share an individual gets increases with his investment in political capital and decreases with the investment of others, the last term in (7) will be larger with higher proportion of corrupt agents and with higher agent's investment in the political capital, and it will be smaller with more political capital accumulated by others.

[10] Ehrlich and Lui (1999) use $s_{it} = 1 + \log\left(\frac{Q_{it}}{Q_t^*}\right)$, which has a disadvantage that if agents invest different amounts of time then the share they receive does not add up to total rents available.

2.4 Consumption

Agents consume all their income at the end of every period.[11] The lifetime consumption of unskilled workers depends on their labor income and the transfers they receive when old. The consumption of the elite on the other hand depends on their investment in human capital, corruption and cost of being corrupt, and their share of political capital relative to the rest of the agents. The elite are homogeneous and we consider only the symmetric equilibria, thus, in equilibrium either all will be corrupt or all will stay honest, this implies that either $d = 0$ or $d = 1$. For $d = 0$, there is no rents appropriation by the elite (no corruption), thus their income will be their wages plus the transfers that they receive when old. On the contrary, $d = 1$ implies that all members of the elite are corrupt and they are appropriating all rents accrued from the natural resources sector. In this case, apart from their labor income, the elite get their share from rents according to their relative political capital, and since the elite divert all rents, there are no transfers.

Given this, total consumption of the elite when there is no corruption is $c^n_{int} = (1 - h_{i1t})H_{i1t} + H_{i2t} + \frac{\alpha Y^R_{2t}}{N}$, where the subscript in indicates that agent i is not corrupt and the superscript n indicates that all other members of the elite are not corrupt. When all members of the elite are corrupt, they divert all rents from the natural resource sector. Each of them consumes $c^c_{ict} = (1 - h_{it})H_{i1t} + (1 - z)\left(H_{i2t}(1 - q_{it}) + \alpha Y^R_{2t}\frac{Q_{i2t}}{nQ_{2t}}\right)$, which in equilibrium is $c^c_{ict} = (1-h_{it})H_{i1t}+(1-z)\left(H_{i2t}(1 - q_{it}) + \frac{\alpha Y^R_{2t}}{n}\right)$, where the subscript ic indicates that agent i is corrupt and the superscript c indicates that all other members of the elite are corrupt. There are three main differences when we compare consumption in the case where there is corruption to the case where there is no corruption. First, with corruption the elite get political rents coming from the natural resource sector. Second, they get lower wage income in the second period as they invest part of their time q_t in accumulating political capital. Third, since there is a monetary cost of being corrupt, a fraction z from their second period income is now eroded.

3 The elite decision problem

The elite's decision making involves two dimensions: first, whether to invest in political capital or not; and second, the time allocation between accumulating capitals (human and political) and working. First, the agent decides whether to invest in political capital or not which depends on the available rents, the share of rents that he gets and the monetary cost. By investing in political capital, he can divert the rents but doing this is associated with a loss of fraction z of his second period income that is dissipated. Then

[11] Although, agents consume their income at the end of every period, the two-period model is useful for our problem as it augment the trade-off between the investment in human and political capitals.

he chooses his optimal time investments in both human and political capitals. If he chooses not to invest in political capital, he will decide time allocation between accumulating human capital and working in the first period, while in the second period he devotes his entire time to working. If he chooses to be corrupt, in the first period, he will decide the optimal time allocation between human capital accumulation and working, and in the second period between political capital accumulation and working.

We proceed to solve the system backwards. In the first step, we obtain the optimal time allocation in both situations, with and without corruption. We start with high-growth (no-corruption) equilibrium. We find the optimal investment in human capital accumulation h_t when the economy is in high-growth equilibrium (i.e. when there is no investment in political capital). Next, we obtain the optimal time allocation in human capital h_t and in political capital q_t in the presence of corruption. Then, in the second step, we compare the level of utility in each of these equilibria and define the existence of equilibrium under parametric ranges. While doing so, we allow individuals to invest differently than each other.

3.1 The high-growth (no corruption) equilibrium

We start with a decision to invest in human capital in the high growth equilibrium which is defined as an equilibrium in which $q_t = 0$. The maximization problem faced by the agent is

$$\begin{aligned} \max_{c,h} \quad & U = c_{1t} + \beta c_{2t} \\ s.t. \quad & H_{i2t} = A h_{it}^{\theta} H_{i1t} \\ & c_{1t} = (1 - h_{it}) H_{i1t} \\ & c_{2t} = H_{i2t} + \frac{\alpha Y_{2t}^R}{N} \end{aligned}$$

together with the constraint $0 \leq h_t \leq 1$. The individual takes output in the natural resource sector Y_{2t}^R as given. The first order conditions yield the high-growth time devoted to human capital investment

$$h_h = (\beta \theta A)^{\frac{1}{1-\theta}} \tag{8-a}$$

While deciding his investment in human capital, the agent faces a trade-off where more time devoted to schooling implies less earnings when young but more earnings when old.

From (2), in high-growth equilibrium, human capital and output grow at

$$1 + g_h = A \left(\beta \theta A\right)^{\frac{\theta}{1-\theta}} \tag{8-b}$$

In the high-growth equilibrium, the growth rate will be faster the more patient individuals are, the more productive the human capital technology is, and the higher the returns to human capital accumulation are.

3.2 Equilibria with corruption

Consider now the maximization problem faced by a member of the elite who chooses to be corrupt when others are corrupt ($q_t > 0, Q_t^- > 0$). The maximization problem is

$$\max_{c,h} \; U = c_{1t} + \beta c_{2t}$$

$$s.t. \quad H_{i2t} = Ah_{it}^{\theta} H_{i1t}$$

$$Q_{i2t} = Bq_{it}^{\gamma} Q_{i1t}$$

$$c_{1t} = (1 - h_{it})H_{i1t}$$

$$c_{2t} = (1 - z)\left(H_{i2t}(1 - q_{it}) + \alpha Y_{2t}^{R} \frac{Q_{i2t}}{nQ_{2t}}\right)$$

together with the constraints $0 \leq h_t \leq 1$ and $0 \leq q_t \leq 1$. The individual takes the output in the natural resource sector Y_{2t}^R and the political capital Q_{2t}^- of others as given, so that we can express utility as

$$U = (1 - h_{it})H_{i1t} + \beta(1 - z)\left[Ah_{it}^{\theta} H_{i1t}(1 - q_{it}) + \frac{a_t \sigma R^{\alpha} Bq_{it}^{\gamma} Q_{i1t}}{Bq_{it}^{\gamma} Q_{i1t} + (n - 1)Q_{2t}^-}\right]$$

where $\sigma = \alpha l^{1-\alpha}$. The first order conditions for h_t and q_t yield

$$h_{it} = \left[\beta\theta A(1 - z)\,(1 - q_{it})\right]^{\frac{1}{1-\theta}} \tag{9-a}$$

$$q_{it} \leq \sigma\gamma R^{\alpha} \frac{Bq_{it}^{\gamma} Q_{i1t}(n - 1)Q_{2t}^-}{(Q_{i2t} + (n - 1)Q_{2t}^-)^2} \frac{a_t}{Ah_{it}^{\theta} H_{i1t}} \tag{9-b}$$

From equation $(9 - a)$, it is evident that there is a trade-off between human and political capital accumulations. A higher investment in political capital would imply spending less time working when old, which in turn implies that the returns to his human capital in the second period would be lower. Thus higher investment in political capital is associated with lower investment in human capital. Further, a higher cost of political participation implies that a higher share of the second period income would be lost which reduces the returns to human capital accumulation so the incentives to invest in it.

In equation $(9{-}b)$, the individual chooses his investment in political capital while taking the investment of others Q_{2t}^- as given. Since the share of rents that an individual gets depends on his investment in political capital relative to the others, the more others invest in political capital, the smaller is the share that the individual gets and thus lower he invests in political capital. Moreover, since agents are symmetric, all agents invest the same time in accumulating political and human capital, $q_{it} = q_t^*$, $h_{it} = h_t^*$, and all have the same political and human capital, $Q_{i2t} = Q_{2t}^- = Q_{2t}$, $H_{i2t} = H_{2t}$. Equation $(9-b)$ then becomes

$$q_t^* \leq \sigma\gamma R^{\alpha}\left(\frac{n-1}{n^2}\right)\frac{a_t}{H_{2t}} \tag{9-c}$$

From equation $(9-c)$, it is evident that the higher the endowment of natural resources is, the higher will be the investment in political capital. The size of the elite n has a negative impact on the investment in political capital. A higher n implies that the rents will be shared amongst many thus the share that individual i gets would be smaller. This condenses the utility of being corrupt, which in turn reduces the incentives to invest in political capital. Thus, increasing equality in access to political participation would reduce the investment in political capital.

Investment in political capital depends on the endowments of natural resources and $(9-c)$ can be written as

$$q_t^* \leq \left(\frac{R}{R^*}\right)^{\alpha} \tag{9-d}$$

where $R^* = \left(\frac{n^2}{a\sigma\gamma(n-1)}\right)^{\frac{1}{\alpha}}$. Since the time allocation is bounded, depending on the endowment of natural resources R, from equation $(9-d)$, we can have an interior solution (where $1 > q_t > 0$) or a corner solution (where $q_t = 1$). Thus there can be two equilibria, a poverty-trap equilibrium (corner solution) when the endowment of natural resources is greater or equal to R^*, and a low-growth equilibrium (interior solution) when the endowment of natural resources is less than R^*.

3.2.1 The poverty-trap equilibrium

The poverty-trap equilibrium arises when the level of natural resources is so high (i.e. $R \geq R^*$) that in the second period agents allocate their entire time in accumulating political capital. For high endowment of natural resources, accruing rents are too high that the incentives to accumulate political capital eliminate incentives to invest in human capital and to work in the manufacturing sector. In this way, investment in political capital completely crowds out investment in human capital. The solution to individual maximization in $(9-d)$ implies that for any $R \geq R^*$

$$q_p = 1 \tag{10-a}$$

$$h_p = 0 \tag{10-b}$$

When the investment in human capital is zero, in equation $(9-c)$, $a_t = \underline{a}$. In the first period, the elite devote no time to human capital accumulation thus they spend their entire time working. In the second period, they devote their entire time for political capital accumulation and do not work at all. As no one invests in human capital, there is no growth, $g_p = 0$.

3.2.2 The low-growth equilibrium

The economy is in low-growth equilibrium when natural resource endowments are not too high to compel agents to invest their entire second period time in accumulating political capital. From $(9-d)$, $q_t < 1$ for any $R < R^*$. Given this, the low-growth investments in human and political capital are

$$q_l = \frac{a\sigma\gamma R^{\alpha}(n-1)}{n^2} \tag{11-a}$$

$$h_l = \left[\beta\theta A(1-z)\left(1 - \frac{a\sigma\gamma R^{\alpha}(n-1)}{n^2}\right)\right]^{\frac{1}{1-\theta}} \tag{11-b}$$

The rate of growth of human capital and output is given by

$$1 + g_l = A\left[\beta\theta A(1-z)\left(1 - \frac{a\sigma\gamma R^{\alpha}(n-1)}{n^2}\right)\right]^{\frac{\theta}{1-\theta}} \tag{11-c}$$

The optimal investment in political capital increases with the endowment of natural resources and decreases with the number of the elite n. A higher resource endowment implies higher rents thus higher incentives to invest in political capital whereas a higher n implies a lower share of rents that every one gets thus lower incentives to invest in political capital. Moreover, since investment in human capital decreases with investment in political capital, a higher endowment of natural resources would imply a lower investment in human capital and a higher n would imply a higher investment in human capital. Thus, in the low-growth equilibrium, a higher endowment of natural resources increases the investment in political capital and reduces the investment in human capital which in turn reduces the rate of growth. Whereas a lower inequality reduces the investment in political capital and increases the investment in human capital which in turn increases the rate of growth. Furthermore, the rate of growth decreases with cost of political participation and returns to political capital, and increases with returns to human capital.

3.3 The decision to accumulate political capital

In the previous section, we presented individual optimization problem and obtained optimal time allocations in both human and political capitals, and the corresponding equilibria. In this section, we look at the existence of those equilibria where an individual decides to invest in political capital while comparing his utility in various scenarios and by taking into account the investments of the others. The individual chooses to invest in political capital if it generates higher welfare than by not investing. Given the different equilibria in previous section, there are three possible scenarios depending on whether others invest an amount $q_t = 0$, $q_t = 1$ or $q_t = q_l$ in political capital. Since, returns to political capital depend on the rents from natural resource sector, the crucial element for the existence of different equilibria would be the endowment of natural resources. Before going into the details of utility comparisons, we suppose $\theta = \frac{1}{2}$. Although, the existence of equilibria can be shown for any value of θ, fixing $\theta = \frac{1}{2}$ helps us to define different thresholds of resource endowment that demarcate different equilibria.

3.3.1 When all others invest $q_t = 0$

Consider first the case when all others $n-1$ are honest, i.e. they do not invest in political capital, $q_t = 0$. Individual i takes the behavior of all others as given and compares the utility when he also does not invest in political capital with that of when he invests. If he does not invest in political capital then there is no rent appropriation and all old agents get equal transfers from the government. If this is the case, the time invested in accumulating human capital is h_h, and the utility of being honest is

$$U^n_{int} = (1-h_h)H_{i1t} + \beta\left(Ah_h^\theta H_{i1t} + \frac{a_t^n \sigma R^\alpha}{N}\right)$$

where $a_t^n = aH_{2t} = aAh_h^\theta H_{1t}$ which requires $H_{1t} > \frac{an}{aAh_h^\theta}$ implying that even when one agent invests in human capital (given the positive inherited human capital), the production technology in the natural resource sector would be $a_t = aH_{2t}$.

Consider now a situation where the individual chooses to be corrupt when every one else is honest. If the agent opts to invest in political capital, since he is the only one to invest, our rent diverting technology implies that he can divert a fraction $\frac{1}{n}$ of rents while the remaining fraction $\frac{n-1}{n}$ is distributed among old population. Moreover, since other agents have no political capital, the optimal investment is an infinitely small amount $q = \varepsilon$, where $\varepsilon \to 0$. While deciding about his investment in human capital, individual takes into account that there is a monetary cost associated with being corrupt, thus the returns that he will get from his investment in human capital would be lower. Given this, the individual maximization problem when he is the only one to invest in political capital is

$$\max_{c,h} U = c_{1t} + \beta c_{2t}$$

$$s.t. \quad H_{i2t} = Ah_{it}^\theta H_{i1t}$$

$$c_{1t} = (1-h_{it})H_{i1t}$$

$$c_{2t} = (1-z)\left[H_{i2t} + \alpha Y_{2t}^R\left(\frac{1}{N}(\frac{n-1}{n}) + \frac{1}{n}\right)\right]$$

The first order conditions yield

$$\widehat{h} = [(1-z)\beta\theta A]^{\frac{1}{1-\theta}} \tag{12}$$

Since part of second period income is now eroded, the optimal investment in human capital is lower than the case of a high-growth equilibrium.

Given the optimal investment in human capital accumulation and taking into account the rents and transfers, the utility of being corrupt when others are honest is

$$U_{ict}^{n} = (1-\widehat{h})H_{i1t} + \beta\,(1-z)\left(A\widehat{h}^{\theta}H_{i1t}(1-\varepsilon) + a_t^c\sigma R^{\alpha}\left(\frac{1}{N}(1-\frac{1}{n}) + \frac{1}{n}\right)\right)$$

where $a_t^c = aH_{2t} = a\frac{A\widehat{h}^{\theta}H_{i1t}+(n-1)Ah_h^{\theta}H_{1t}}{n}$.

Proposition 1 $\forall R \leq \overline{R}$, *there exists a high-growth equilibrium such that no one invests in political capital.*

Proof. See Appendix 1. ■

For the endowment of natural resources less than are equal to $\overline{R}$, returns to political capital are so low that no one has incentive to invest in political capital. Thus for $R \leq \overline{R}$, the economy is in high-growth equilibrium with no corruption and higher human capital, where

$$\overline{R} \equiv \left(\frac{Nn^2\left(1-(1-z)^2\right)}{2a\sigma\left[(1-z)\left(N+n-1\right)(n-z)-n^2\right]}\right)^{\frac{1}{\alpha}} \tag{13}$$

Whereas, for the endowment of natural resources greater than $\overline{R}$, the individual invests in political capital even though all others do not, implying that a high-growth is not an equilibrium for any $R > \overline{R}$. The threshold $\overline{R}$ depends on the cost of political participation z, and the number of the elite n. It increases with z; the higher the cost of political participation is, the higher will be $\overline{R}$, thus, the higher will be the range of R where a high-growth equilibrium exists. If there is zero cost of political participation, $z = 0$, then $\overline{R} = 0$ and the high-growth equilibrium does not exist. This implies that with zero cost, for any endowment of natural resources it will always be profitable to invest in political capital. Whereas, a non-zero cost of political participation implies that the individual loses a part of his legal income as well; thus, when there is low endowment of natural resources, the rents are too small to cover the expected losses caused by z and the individual has no incentive to invest in political capital. The threshold $\overline{R}$ also increases with the number of the elite n.[12] The lower inequality in access to education and political process is (a higher n), the higher will be the range of natural resource endowments for which there is a high-growth equilibrium. Thus, reducing inequality in access to education and political process would increase the range of R for which a high-growth equilibrium exists.

[12] This is always true for any $n \geq 3$ and $N \geq n+1$.

3.3.2 When all others invest $q_t = 1$

When all others are corrupt and they invest $q_t = 1$ time in political capital accumulation, if agent i also invests in political capital, in equilibrium $q_t = 1$ and $h_t = 0$. In the first period, they invest their entire time working in the manufacturing sector and in the second period, they invest their entire time accumulating political capital and there will be no production in the manufacturing sector.[13] Since all agents are corrupt, they divert all rents and in equilibrium every one gets $\frac{1}{n}$ share of it. Given this the utility of being corrupt when every one else is corrupt is

$$U_{ict}^c = H_{i1t} + \frac{\beta(1-z)\underline{a}\sigma R^\alpha}{n}$$

If the individual chooses not to be corrupt when all other agents are corrupt, he solves his maximization problem by taking into account the fact that all others are corrupt. He invests $h_t = h_h$ in human capital accumulation. Since all $n-1$ agents are corrupt, they divert $\frac{n-1}{n}$ share of rents from the natural resource sector and the remaining $\frac{1}{n}$ share of rents is distributed among all old agents. Given, this the utility of being honest when all others are corrupt is

$$U_{int}^c = (1-h_h)H_{i1t} + \beta\left(Ah_h^\theta H_{i1t} + \frac{a_t^n \sigma R^\alpha}{N}\left(\frac{n-1}{n}\right)\right)$$

where $a_t^n = aH_{2t}$. Since, other $n-1$ do not invest in human capital, the average stock of human capital will be, $H_{2t} = \frac{Ah_h^\theta H_{i1t}}{n}$.

Proposition 2 $\forall R \geq \widehat{R}$, *there exists a poverty-trap equilibrium such that all members of the elite invest* $q_t = 1$ *in political capital.*

Proof. See Appendix 2. ■

The endowments of natural resources are so high that all members of the elite devote their entire second period time to accumulating political capital and there is no production in the manufacturing sector. Thus for any $R \geq \widehat{R}$, there exists a poverty-trap equilibrium, where

$$\widehat{R} \equiv \left(\frac{\beta A^2 H_{1t} N n^2}{2\sigma\left[2Nn\underline{a}(1-z) - \beta A^2 H_{1t} a(n-1)\right]}\right)^{\frac{1}{\alpha}} \tag{14}$$

[13] Since, output in the manufacturing sector depends on the stock of human capital, the output of the manufacturing sector will be different for the first generation (as agents inherit a positive stock of human capital) than the subsequent generations (there is no human capital). The inherited human capital of the individual of first generation is $H_{2t-1} = H_{i11} > 0$, and since agents invest their entire time in working when young (no human capital accumulation), the wage that the elite will get is H_{i11}. Given the wage in the first period, the utility of being corrupt will be $U_{ic1}^c = H_{i11} + \beta(1-z)\frac{\alpha\underline{a}l^{1-\alpha}R^\alpha}{n}$. Whereas, in the subsequent periods, since there is no human capital accumulation ($H_{i21} = 0$), there is no production in the manufacturing sector, thus no wage. And since there is no wage income, the only income of the elite will be the rents that they appropriate from the natural resource sector. The utility of being corrupt in this case will be $U_{ic2}^c = \beta(1-z)\frac{\alpha\underline{a}l^{1-\alpha}R^\alpha}{n}$.

Whereas, for the endowment of natural resources is lower than $\widehat{R}$, the agent do not invest in political capital even when every one else is investing.

From the individual maximization in the previous section, we have that the poverty-trap equilibrium exists (i.e. it is optimal to invest $q_t = 1$) for any endowment of natural resources greater or equal to R^*. In this way, for the existence of poverty-trap equilibrium, we have two conditions, i.e. from the optimization problem, we have $R \geq R^*$ and from the above incentive constraint, we have $R \geq \widehat{R}$. By comparing these two thresholds of R, we find that R^* is greater than $\widehat{R}$ for any cost of political participation less than $\overline{z}$, where

$$\overline{z} \equiv 1 - \frac{\beta a A^2 H_{1t}(n-1)(N\gamma + 2)}{4Nn\underline{a}}$$

For the rest of the paper, we assume that the cost of political participation is not too large to erode the incentives to be corrupt, i.e. $R^* \geq \widehat{R}$.

Assumption A1: $z \leq \overline{z}$.

Assumption A1 implies that when it is optimal to invest $q_t = 1$ time in political capital accumulation, the poverty-trap equilibrium always exists. Note that for any $z > \overline{z}$, there will be a poverty-trap equilibrium for any $R \geq \widehat{R}$.

3.3.3 When all others invest $q_t = q_l$

When all others are corrupt and they invest $q_t = q_l$ in political capital, if the agent i also invests in political capital, in equilibrium $q_t = q_l$ and $h_t = h_l$. Since all agents are corrupt, they divert all rents and in equilibrium every one gets $\frac{1}{n}$ share of it. Given this the utility of being corrupt when every one else is corrupt is

$$U^c_{ict} = (1 - h_l)H_{i1t} + \beta(1 - z)\left[Ah_l^\theta H_{i1t}(1 - q_l) + a^c_t \sigma R^\alpha \left(\frac{1}{n}\right)\right]$$

where $a^c_t = \max\{\underline{a}, aH_{2t}\}$. We have $H_{2t} = Ah_l^\theta H_{1t}$, and aH_{2t} is greater than $\underline{a}$ for $h_l > \underline{h} \equiv \left(\frac{\underline{a}}{aAH_{1t}}\right)^{\frac{1}{\theta}}$. By comparing h_l with $\underline{h}$, we have $h_l \geq \underline{h}$ for all $R \leq \widetilde{R} \equiv \left(\frac{n^2}{a\sigma\gamma(n-1)}\left(1 - \frac{1}{1-z}\left(\frac{\underline{h}}{h_h}\right)^{1-\theta}\right)\right)^{\frac{1}{\alpha}}$, which implies that $a^c_t = aAh_l^\theta H_{1t}$ for all $R \leq \widetilde{R}$ and $a^c_t = \underline{a}$ for all $R > \widetilde{R}$. U^c_{ict} is piecewise concave function with respect to R in the range $R \in (0, R^*)$ with a possible kink at $R = \widetilde{R}$.[14]

[14] The detailed derivations of the function U^c_{ict} are given in Appendix 3.

If the individual chooses not to be corrupt when all others are corrupt, he solves his maximization problem by taking into account the fact that all others are corrupt. Since all $n-1$ agents are corrupt, they divert $\frac{n-1}{n}$ share of rents from natural resource sector and the remaining $\frac{1}{n}$ share of rents is distributed among all old agents. Given this, the utility if being honest when all others are corrupt is

$$U_{int}^{c} = (1-h_h)H_{i1t} + \beta\left[Ah_h^{\theta}H_{i1t} + \frac{a_t^n \sigma R^{\alpha}}{N}\left(\frac{n-1}{n}\right)\right]$$

where $a_t^n = aH_{2t} = a\left(\frac{Ah_h^{\theta}H_{i1t}+(n-1)Ah_l^{\theta}H_{1t}}{n}\right)$ and U_{int}^{c} is a concave function with respect to R in the range $R \in (0, R^*)$.[15]

By comparing the both utilities, there exists $\underline{N} \geq 0$ such that $U_{ict}^{c} = U_{int}^{c}$ at $R = \underline{R}$ and for any $R > \underline{R}$, $U_{ict}^{c} > U_{int}^{c}$. In the remaining of the paper, we consider $N > \underline{N}$, where

$$\underline{N} \equiv \frac{(1-z-x)(n-1)\left(x^{\frac{-\theta}{1-\theta}} + n - 1\right)}{n(1-z)\left(\gamma(n-1)(1-\theta)\left(x - x^{\frac{-\theta}{1-\theta}}\right) + (1-z-x)\right)}$$

where $x = \left(\frac{h}{h_h}\right)^{1-\theta}$. Graphically,

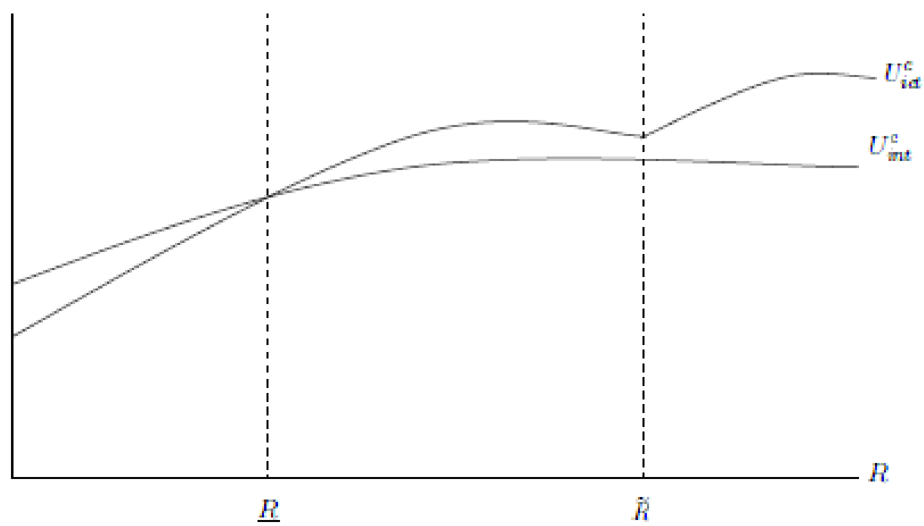


Figure 1

Combining this with our results from the individual optimization problem which implies that the low-growth equilibrium exists for $R < R^*$, we have our results in Proposition 3.

[15] The detailed derivations of the function U_{int}^{c} are given in Appendix 3.

Proposition 3 *$\forall N > \underline{N}$, there exists a unique $\underline{R}$ such that a low-growth equilibrium exists for $R^* > R \geq \underline{R}$ where all members of the elite invest $q_t = q_l$ in political capital.*

Proof. See Appendix 3. ■

The low-growth equilibrium exists for the intermediate range of the natural resource endowments. It does not exist for a lower endowment of natural resources as lower endowment would imply low rents, thus, no investment in political capital, and it does not exist for a higher endowment of natural resources as higher endowment would imply high rents, thus, investing entire second period time in political capital.[16]

4 Natural resource thresholds and equilibria

4.1 Equilibrium configurations

In this section we use our results from the previous section to compare different thresholds of natural resource endowments that characterize the three possible equilibria. Our results imply that

- the high-growth equilibrium exists for all $R \leq \overline{R}$.
- the low-growth equilibrium exists for all $\underline{R} \leq R < R^*$.
- the poverty-trap exists for all $R \geq R^*$.

From the Assumption A1, we know that R^* is always greater than $\widehat{R}$. By comparing $\overline{R}$ with R^*, we find that $\overline{R} > R^*$ for all z greater than $\widehat{z}$, where

$$\widehat{z} \equiv 1 - \frac{-(n-1)(N+n-1) + \sqrt{(n-1)^2(N+n-1)^2 + \left[2N + (n-1)(2+N\gamma)\right]\left(N\gamma(n-1) + 2n^2\right)}}{2N + (n-1)(2+N\gamma)}$$

By comparing $\underline{R}$ with R^*, we know from Proposition 3 that $R^* > \underline{R}$. Thus, for $z > \widehat{z}$ we know that $\overline{R}$ is greater than R^* and since R^* is always greater than $\underline{R}$, we have $\overline{R} > \underline{R}$. Whereas, for cost of political participation less than $\widehat{z}$, $R^* > \overline{R}$ and also $R^* > \underline{R}$, since, we do not have expression for $\underline{R}$, we do not know whether $\overline{R}$ is greater than or less than $\underline{R}$.

Proposition 4 *$\forall z > \widehat{z}$; there is a unique poverty-trap equilibrium $\forall R > \overline{R}$, there is a unique high-growth equilibrium $\forall R < \underline{R}$, whereas for $\overline{R} \geq R \geq \underline{R}$, there are multiple equilibria, where for $R^* > R \geq \underline{R}$, high-growth and low-growth equilibria coexist and for $\overline{R} \geq R \geq R^*$, high-growth and poverty-trap equilibria coexist.*

[16] We are unable to find an expression for $\underline{R}$, thus, we can not make comparative statics of $\underline{R}$ with respect to cost of political participation z or inequality in access to education and political process n.

Proposition 4 depicts the case of quality institutions where costs of political participation are higher. For low and high endowments of natural resources, there is a unique high-growth and a poverty-trap equilibrium, respectively. In Figure 2, we depict our results of better quality institutions from Proposition 4

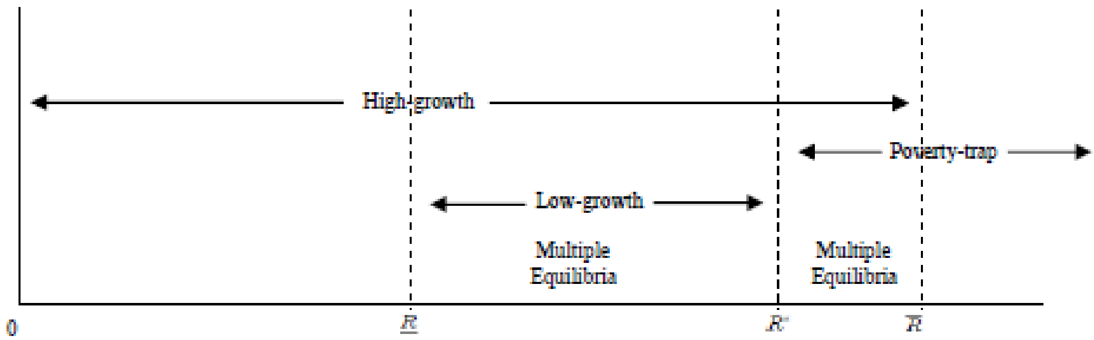


Figure 2

Existence of multiple equilibria for the intermediate levels of natural resource endowments imply that the relationship between natural resources and resource-curse is non-monotonic. High-growth equilibrium coexist with both low-growth and poverty-trap equilibria. Thus, we can have countries with identical endowments of natural resources experiencing two different growth regimes; a high-growth regime with no corruption and high investment in human capital, or a poverty-trap with no growth, no investment in education and high corruption.

Proposition 5 *$\forall z < \widehat{z}$ and $\overline{R} > \underline{R}$, there is a unique poverty-trap equilibrium $\forall R > R^*$, there is a unique high-growth equilibrium $\forall R < \underline{R}$, there is a unique low-growth equilibrium for $R^* > R \geq \overline{R}$, whereas for $\overline{R} > R \geq \underline{R}$, there are multiple equilibria, where high-growth and low-growth equilibria coexist.*

Proposition 5 depicts the case of bad institutions where cost of political participation is low. There are three main differences from the case with better quality institutions; first, with low cost of political participation, high-growth and poverty-trap equilibria do not coexist. Second, the range of natural resource endowments where there is a unique poverty-trap equilibrium is higher i.e. it exists for any $R > R^*$ which is smaller than $\overline{R}$ in Figure 2. Third, with low quality institutions for certain levels of R, there is a unique low-growth equilibrium while with quality institution it always coexisted with a high-growth equilibrium. In Figure 3, we depict our results of low quality institutions presented in Proposition 5

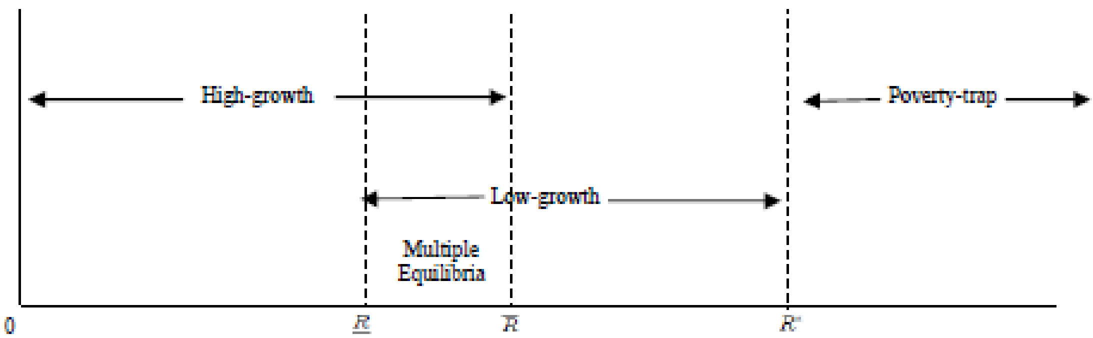


Figure 3

Figure 3 depicts the case of low quality institutions. For low endowment of natural resources, there is a unique high-growth equilibrium and for high endowments of natural resources, there is a unique poverty-trap equilibrium. For the intermediate levels, the high-growth and the low-growth equilibria coexist for $R \in (\underline{R}, \overline{R})$, and for any $R^* > R > \overline{R}$, there is a unique low-growth equilibrium.

With $z < \widehat{z}$, there can be a possibility where $\overline{R}$ is lower than $\underline{R}$. In Figure 4, we put our results of Proposition 5 with $\overline{R} < \underline{R}$. For low endowment of natural resources only the high-growth equilibrium exists while for high endowment of natural resources only the poverty-trap equilibrium exists. For the intermediate ranges, the low-growth equilibrium exists for $R \in (\underline{R}, R^*)$. In the range of endowments $R \in (\overline{R}, \underline{R})$, there is a possibility of no symmetric equilibrium.

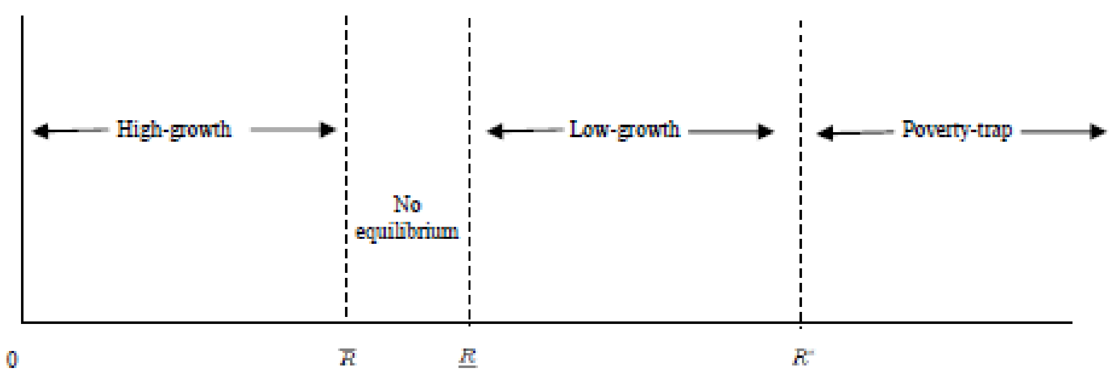


Figure 4

The no-equilibrium scenario can be attributed to the fact that we are looking at only the symmetric equilibrium, i.e. either all members of the elite invest in political capital or no one invests. Note that for the stock of natural resources less or equal to $\overline{R}$, the proportion of the elite investing in political capital is zero, $p_t = 0$ and for any stock of natural resources greater or equal to $\underline{R}$, $p_t = 1$. Thus, moving from $\overline{R}$ to $\underline{R}$, the proportion of the elite investing in political capital jumps from zero to one. Although, we are unable to show it analytically, our intuition is that by allowing for an asymmetric equilibrium, there can be an equilibrium where a proportion of the elite would be investing in political capital.

In the above equilibrium configurations, the thresholds of natural resource endowments that demarcate different equilibria are endogenous and depend on model parameters. The policies that affect these parameters can then shift the economy to a different equilibrium. The region where the high-growth equilibrium exists depends on inequality in access to education and political participation, n and the quality of institutions, z. Since $\overline{R}$ depends on the cost of political participation and the number of elite, the multiple equilibria are determined by both z and n. Threshold $\overline{R}$ increases with n, which is to say that increasing access to education and political participation would increase the region where the high-growth is equilibrium. Whereas, the threshold R^* decreases with n implying that increasing access to education and political participation would reduce the region of natural resource endowments where the poverty-trap is equilibrium.

Another crucial element in the determination of multiple equilibria is the cost of political participation. When there is no cost involved in rent seeking i.e. $z = 0$, then $\overline{R} = 0$, and the high-growth equilibrium does not exist. The intuition behind this is very straight-forward, natural resources create rents and if there is no cost involved in directing these rents to one's own pockets, even an infinitely small amount of investment in political capital would be sufficient to extract all rents. When there is no cost of political participation, the utility of being corrupt will always be higher than the utility of being honest even if all other agents were honest. This is analogous to say that in the absence of quality institutions, natural resources can only be curse but if the institutions are better (i.e. z is high), it is possible to enjoy a high-growth with high endowments of natural resources.

4.2 Comparative statics

In this section, we summarize our results and look at the features of different equilibria. Table 1 summarizes our results for investments in both human and political capitals, the growth rate and the aggregate output. Natural resource endowments demarcate regions where each of the three equilibria exists. It is evident from the third row of the Table 1 that resource abundance increases the incentives to invest in political capital, and since high investment in political capital crowds out investment in human capital, a higher endowment of natural resources reduces investments in human capital (in second row). Moreover, since human capital is the only productive capital, reducing investment in it would reduce the rate of growth. When endowments are sufficiently high, the elite when young do not invest in schooling, and when old, they invest their entire time in accumulating political capital. Consequently, there is no human capital and no production in the manufacturing sector.

Table 1: Characterization of equilibria

	High-growth	Low-growth	Poverty-trap
Natural resources	$R \leq \overline{R}$	$\underline{R} \leq R < R^*$	$R \geq R^*$
Human capital h_t	$h_h = (\beta\theta A)^{\frac{1}{1-\theta}}$	$h_l = \left[\beta\theta A(1-z)\left(1-\left(\frac{R}{R^*}\right)^{\alpha}\right)\right]^{\frac{1}{1-\theta}}$	$h_p = 0$
Political capital q_t	$q_h = 0$	$q_l = \left(\frac{R}{R^*}\right)^{\alpha}$	$q_p = 1$
Growth of H	$g_h = A\,(\beta\theta A)^{\frac{\theta}{1-\theta}}$	$g_l = A\left[\beta\theta A(1-z)\left(1-\left(\frac{R}{R^*}\right)^{\alpha}\right)\right]^{\frac{\theta}{1-\theta}}$	$g_p = 0$
Aggregate output	$Y_{ht} = nH_{1t}[1+(1+g_h)(1-\beta\theta$ $+\frac{a l^{1-\alpha} R^{\alpha}}{n^2}]$	$Y_{lt} = nH_{1t}[1+(1+g_l)((1-\beta\theta(1-z)$ $(1-\left(\frac{R}{R^*}\right)^{\alpha}) + \frac{a l^{1-\alpha} R^{\alpha}}{n^2})]$	$Y_{p1} = nH_{11} + \underline{a}\overline{l}^{1-\alpha} R^{\alpha}$ $Y_{p2} = \underline{a}\overline{l}^{1-\alpha} R^{\alpha}$

Apart from affecting the existence of different equilibria, natural resources have an impact on aggregate output as well. It affects aggregate output through three channels. First, there is a direct positive impact as high endowment of natural resources implies higher production in the natural resources sector implying higher aggregate output. Second, there is an indirect negative effect arising from the investment in political capital. Higher endowment of natural resources increases the investment in political capital that reduces time invested in human capital as well as time spent working. Thus, with higher endowment of natural resources, output in the manufacturing sector and aggregate output would decline. Third, an indirect negative effect arises through the rate of growth. Higher endowment of natural resources reduces the rate of growth which results in lower output in both manufacturing and natural resources sectors. The overall impact of natural resource abundance on aggregate output is ambiguous and depends on the type of equilibrium. In the high-growth equilibrium, column one of the fifth row, the aggregate output increases with R, hence resources are 'blessings'. In the high-growth equilibrium the investment in human capital and thus the rate of growth would increase with R. Furthermore, a higher R implies a higher production in the natural resource sector and aggregate output. In the case of a low-growth equilibrium, the effect of R is ambiguous. A higher R increases output in the natural resource sector, but reduces the time spent working in the manufacturing sector and investment in human capital, thus the rate of growth.

5 Conclusion

Recent empirical evidence suggests that relatively resource rich countries tend to have lower economic growth, higher corruption and lower level of education. In this paper, we provide a theory behind this evidence where abundance of natural resources affects both corruption and education, which in turn determines the rate of growth. We have developed an endogenous growth model for an economy divided in two classes; the elite and workers. The former is a privileged class who has access to both education and the political process. Apart from the industrial sector, there is a natural resource sector that creates rents. The rents from natural resource sector accrue to the government, which attracts rent seeking. Profitable rent seeking requires time investment in political capital accumulation, which crowds out time invested in education.

Our predictions in this paper are in line with the empirical findings that the natural resources curse operates through the crowding out of productive capital and rent seeking. Furthermore, the relationship between abundance of natural resource and the resource curse is non-monotonic. We find that depending on the natural resource endowments, there can be three different growth regimes. There are endogenous thresholds of natural resource endowments that demarcate different equilibria. For low endowment of natural resources, there is a unique high-growth equilibrium with faster growth, higher education attainment, and no corruption, while for high endowment of natural resources, there is a unique poverty-trap equilibrium with no growth, no education attainment and very high corruption. In the intermediate ranges, there are multiple equilibria.

The thresholds are endogenous and crucially depend on inequality in access to human and political capital accumulation and on the monetary cost associated with corruption. Increasing access to education and political participation would increase the ranges of resource endowments where the high-growth equilibrium exists. Institutions play pivotal role for the determination of different growth regimes. For better quality institutions (i.e. higher cost of political participation), the range of natural resource abundance where high-growth equilibrium exists would be higher. Thus, for higher cost of political participation there are multiple equilibria, where for high abundance of natural resources, high-growth and poverty-trap equilibria coexist, and for the intermediate ranges of natural resource abundance, high-growth and low-growth equilibria coexist. Whereas, decrease in inequality in access to education and political process increases the range of natural resources where the high-growth is in equilibrium and decreases the rage of natural resources where the poverty-trap is an equilibrium.

There is an important limitation in our model concerning the technology of natural resource sector. The assumption that the natural resource sector employs only unskilled workers can be suitable for some types of natural resources but may not be for the others. Second, this limits our analysis to see the impact of natural resources on income inequality. Clearly, when the society is segmented in two classes, abundance of natural resources may have redistributive implications as well. Allowing for more flexible settings may help to see the redistributive implications of natural resources.

References

[1] Aldave, I., & Garcia-Peñalosa, C. (2009). "Education, Corruption and the Natural Resource Curse," Working Papers. Banco Central de Reserva del Perú.

[2] Auty, Richard M. (2001). "The Political Economy of Resource-Driven Growth," *European Economic Review,* 45, May, 839-846.

[3] Baland, J.-M., Francois, P., (2000). "Rent-seeking and resource booms," *Journal of Development Economics,* 61, 527–542.

[4] Bevan, D., P. Collier and J.W. Gunning (1999). The Political Economy of Poverty, Equity and Growth: Nigerian and Indonesia, Oxford University Press, Oxford.

[5] Birdsall, N., Pinckney, T and Sabot, R. (2001). "Natural Resources, Human Capital and Growth," in Resource Abundance and Economic Growth, R. M. Auty, ed., Oxford University Press.

[6] Bravo-Ortega C. and J. De Gregorio, (2005). "The Relative Richness of the Poor? Natural Resources, Human Capital and Economic Growth," Policy Research Working Paper Series 3484, The World Bank.

[7] Bulte, EH., Damania, R., and Deacon, R T., (2005). "Resource Intensity, Institutions, and Development", *World Development*, vol.33(7), 1029–1044.

[8] D'Albis, H., and Ambec, S. (2010). "Fair intergenerational sharing of a natural resource." *Mathematical Social Sciences*, 59(2), 170-183.

[9] Dasgupta, P. and Heal, G.M. (1974). "The optimal depletion of exhaustible resources," *Review of Economic Studies*, Symposium, 3-28.

[10] Ehrlich, I. and Lui, F.T. (1999). "Bureaucratic Corruption and Endogenous Economic Growth," *Journal of Political Economy*, 107: S270-S293.

[11] Eicher T., García-Peñalosa, C and van Ypersele, T. (2009). "Education, Corruption and the Distribution of Income," forthcoming *Journal of Economic Growth.*

[12] Glaeser, E.L., R. La Porta, Lopez-de-Silanes, F. and Shleifer, A. (2004). "Do Institutions Cause Growth?" *Journal of Economic Growth*, 9: 271-303.

[13] Gylfason, T. (2001). "Natural Resources, Education and Economic Development", *European Economic Review*, 45: 847-859.

[14] Gylfason, T., T.T. Herbertsson, G. Zoega, (1999). "A Mixed Blessing", *Macroeconomic Dynamics*, 3: 204-225.

[15] Lane, P.R., Tornell, A., (1996). "Power, growth and the voracity effect," *Journal of Economic Growth,* 1, 213–241.

[16] Matsuyama, K., (1992). "Agricultural productivity, comparative advantage, and economic growth." *Journal of Economic Theory* 58, no. 2: 317-334.

[17] Mitra, T. (2002). "Intertemporal equity and efficient allocation of resources," *Journal of Economic Theory*, 107, 356-376.

[18] Mehlum, H, Moene, K., O, and Torvik, R. (2006). "Cursed by resources or institutions?," *World Economy*, August, 1117-1031.

[19] Robinson, J., Torvik, R., & Verdier, T. (2006). "Political foundations of the resource curse," *Journal of Development Economics*, 79(2), 447-468.

[20] Ross. (2001), Timber Booms and Institutional Breakdown in Southeast Asia. New York: Cambridge University Press.

[21] Sachs, J.D., Warner, A.M., (2001). "Natural Resources and Economic Development: The Curse of Natural Resources." *European Economic Review,* 45:827-838.

[22] Sachs, J.D., Warner, A.M., (1999). "The Big Push, Natural resource Booms and Growth," *Journal of Development Economics*, 59: 43-76.

[23] Sachs, J.D., Warner, A.M., (1995). "Natural resource abundance and economic growth," NBER Working Paper No. 5398.

[24] Sala-i-Martin, X. and Subramanian, A. (2003). "Addressing the natural resource curse: An illustration from Nigeria," *NBER Working Paper* 9804, Cambridge, Mass.

[25] Sarraf, M. and Jiwanji, M. (2001). "Beating the resource curse. The case of Botswana, Environment Department," World Bank, Washington D.C.

[26] Solow, R.M. (1974). "Intergenerational equity and exhaustible resources," *Review of Economic Studies*, Symposium, 29-45.

[27] Stiglitz, J. (1974). "Growth with exhaustible natural resources: efficient and optimal growth paths," *Review of Economic Studies*, Symposium, 123-137.

[28] Stijn, J, P. (2006). "Natural Resource Abundance and Human Capital Accumulation," *World Development*, 34(6), 1060-1083.

[29] Svensson, J. (2005). "Eight Questions about Corruption," *Journal of Economic Perspectives*, 19(3), 19-42.

[30] Torvik, R. (2001). "Learning by doing and the Dutch disease," *European Economic Review,* 45, 285–306.

[31] Torvik, R. (2002). "Natural resources, rent seeking and welfare," *Journal of Development Economics*, 67(2), 455-470.

Appendix 1

In this Appendix, we prove the existence of a high-growth equilibrium. The high-growth equilibrium exists if the utility of being honest is greater than the utility of being corrupt when every one else is honest. The utility of being honest is

$$U_{int}^{n}=(1-h_h)H_{i1t}+\beta\left(Ah_h^{\theta}H_{i1t}+\frac{a_t\sigma R^{\alpha}}{N}\right)$$

where $a_t=aAh_h^{\theta}H_{1t}$.

The utility of being corrupt is

$$U_{ict}^{n}=(1-\widehat{h})H_{i1t}+\beta\left(1-z\right)\left(A\widehat{h}^{\theta}H_{i1t}(1-\varepsilon)+aH_{2t}\sigma R^{\alpha}\left(\frac{1}{N}(1-\frac{1}{n})+\frac{1}{n}\right)\right)$$

where $H_{2t}=\frac{H_{2it}+(n-1)H_{2t}^{-}}{n}$, since all others are honest, they invest h_h time in human capital accumulation, $H_{2t}^{-}=Ah_h^{\theta}H_{1t}$, and $H_{2t}=\frac{A\widehat{h}^{\theta}H_{i1t}+(n-1)Ah_h^{\theta}H_{1t}}{n}$. When $\varepsilon\to 0$, the utility of being corrupt when all others are honest is

$$U_{ict}^{n}=(1-\widehat{h})H_{1t}+\beta\left(1-z\right)\left(1+\frac{AH_{1t}a\sigma R^{\alpha}\left(N+n-1\right)}{Nn^2}\left(\widehat{h}^{\theta}+(n-1)h_h^{\theta}\right)\right)$$

Given both utilities, the existence of high-growth equilibrium requires

$$U_{int}^{n}(H_{i1t},h_h,a_t^{n},R)\geq U_{ict}^{n}(H_{i1t},\widehat{h},a_t^{c},R)$$

$$h_h^{\theta}\left(1-\theta+\frac{a\sigma R^{\alpha}}{N}\right)\geq\widehat{h}^{\theta}(1-z)\left(1-\theta+\frac{a\sigma R^{\alpha}\left(N+n-1\right)}{Nn^2}\right)+h_h^{\theta}(1-z)\frac{a\sigma R^{\alpha}\left(N+n-1\right)\left(n-1\right)}{Nn^2}$$

$$h_h^{\theta}\left(1-\theta+\frac{a\sigma R^{\alpha}}{Nn^2}\left(n^2-\left(1-z\right)\left(N+n-1\right)\left(n-1\right)\right)\right)\geq\widehat{h}^{\theta}(1-z)\left(1-\theta+\frac{a\sigma R^{\alpha}\left(N+n-1\right)}{Nn^2}\right)$$

$$Nn^2(1-\theta)+a\sigma R^{\alpha}\left(n^2-\left(1-z\right)\left(N+n-1\right)\left(n-1\right)\right)\geq\left(\frac{\widehat{h}}{h_h}\right)^{\theta}(1-z)\left(Nn^2(1-\theta)+a\sigma R^{\alpha}\left(N+n-1\right)\right)$$

By substituting in for $\widehat{h}$ and h_h, we get

$$Nn^2(1-\theta)+a\sigma R^{\alpha}\left(n^2-\left(1-z\right)\left(N+n-1\right)\left(n-1\right)\right)\geq(1-z)^{\frac{1}{1-\theta}}\left(Nn^2(1-\theta)+a\sigma R^{\alpha}\left(N+n-1\right)\right)$$

$$Nn^2(1-\theta)(1-(1-z)^{\frac{1}{1-\theta}})\geq a\sigma R^{\alpha}\left((1-z)^{\frac{1}{1-\theta}}\left(N+n-1\right)+\left(1-z\right)\left(N+n-1\right)\left(n-1\right)-n^2\right)$$

By setting $\theta=\frac{1}{2}$ and solving for R, we get

$$R\geq\overline{R}\equiv\left(\frac{Nn^2(1-(1-z)^2)}{2a\sigma\left[\left(1-z\right)\left(N+n-1\right)\left(n-z\right)-n^2\right]}\right)^{\frac{1}{\alpha}}$$

Appendix 2

In this Appendix, we prove the existence of a poverty-trap equilibrium. The poverty-trap equilibrium exists if the utility of being corrupt is greater than the utility of being honest when every one else is corrupt. This requires

$$U^c_{ict}(H_{i1t}, \underline{a}, R) \geq U^c_{int}(H_{i1t}, h_h, a^n_t, R)$$

$$H_{i1t} + \frac{\beta(1-z)\underline{a}\sigma R^\alpha}{n} \geq (1-h_h)H_{i1t} + \beta A h_h^\theta H_{i1t}\left(1 + \frac{a\sigma R^\alpha(n-1)}{Nn^2}\right)$$

$$\frac{(1-z)\underline{a}\sigma R^\alpha}{n} \geq A h_h^\theta H_{i1t}\left(1 - \theta + \frac{a\sigma R^\alpha(n-1)}{Nn^2}\right)$$

By substituting in for h_h^θ

$$\frac{(1-z)\underline{a}\sigma R^\alpha}{n} \geq A H_{i1t}\left(\beta\theta A\right)^{\frac{\theta}{1-\theta}}\left(1 - \theta + \frac{a\sigma R^\alpha(n-1)}{Nn^2}\right)$$

By setting $\theta = \frac{1}{2}$ and since in symmetric case $H_{i1t} = H_{1t}$, we have

$$R \leq \widehat{R} \equiv \left(\frac{\beta A^2 H_{1t} N n^2}{2\sigma\left[2Nn\underline{a}(1-z) - \beta A^2 H_{1t} a(n-1)\right]}\right)^{\frac{1}{\alpha}}$$

Appendix 3

In this appendix, we prove the existence of a low-growth equilibrium. The low-growth equilibrium exists if and only if the following incentive compatibility constraint is satisfied

$$U^c_{ict}(H_{i1t}, h_l, q_l, a^c_t, R) \geq U^c_{int}(H_{i1t}, h_h, a^n_t, R)$$

$$U^c_{ict} = H_{i1t}\left[1 + \beta A h_l^\theta (1-z)(1-q_l)(1-\theta)\right] + \frac{\beta(1-z)a^c_t \sigma R^\alpha}{n}$$

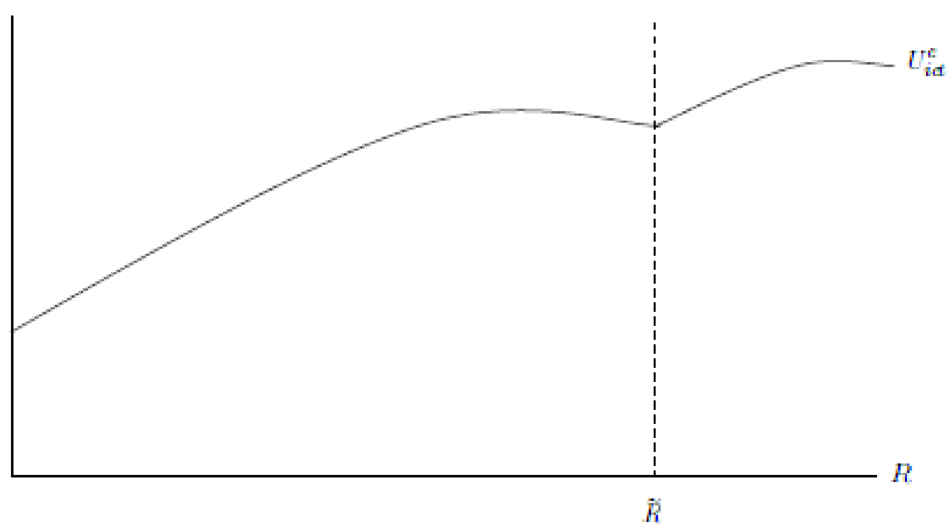
where $a^c_t = \begin{cases} aAh_l^\theta H_{1t} & \forall h_l \geq \underline{h} \\ \underline{a} & \forall h_l < \underline{h} \end{cases}$

Note that $h_l \geq \underline{h}$ for all $R \leq \widetilde{R} \equiv \left(\frac{n^2}{a\sigma\gamma(n-1)}\left(1 - \frac{1}{1-z}\left(\frac{\underline{h}}{h_h}\right)^{1-\theta}\right)\right)^{\frac{1}{\alpha}}$, which implies that $a^c_t = aAh_l^\theta H_{1t}$ for all $R \leq \widetilde{R}$ and $a^c_t = \underline{a}$ for all $R > \widetilde{R}$.

Furthermore, from the agent's optimization problem, we know that the low-growth equilibrium exists for any $R < R^*$. Also, we know that for any $R \geq R^*$, $h_l = 0$. By comparing $\widetilde{R}$ and R^*; since h_l decreases with R, and since at $R = \widetilde{R}$, $h_l = \underline{h} > 0$ and at $R = R^*$, $h_l = 0$, we have $R^* > \widetilde{R}$. Thus the utility of being corrupt is $U^c_{ict}(a^c_t = aAh_l^\theta H_{1t})$ for $R < \widetilde{R}$ and $U^c_{ict}(a^c_t = \underline{a}\ )$ for $\widetilde{R} < R < R^*$.

$$U^c_{ict} = \begin{cases} H_{i1t}\left[1 + \beta A h_l^\theta (1-z)(1-q_l)(1-\theta)\right] + \frac{\beta(1-z)aAh_l^\theta H_{1t}\sigma R^\alpha}{n} & \forall R \leq \widetilde{R} \\ \\ H_{i1t}\left[1 + \beta A h_l^\theta (1-z)(1-q_l)(1-\theta)\right] + \frac{\beta(1-z)\underline{a}\sigma R^\alpha}{n} & \forall R > \widetilde{R} \end{cases}$$

U^c_{ict} is piecewise concave with respect to R in the range $R \in (0, R^*)$. In the following graph, we plot the utility of being corrupt, which is piecewise concave,



Consider first for $\forall R \leq \widetilde{R}$

$$U^c_{ict} = H_{i1t}\left[1 + \beta A h_l^\theta (1-z)(1-q_l)(1-\theta)\right] + \frac{\beta(1-z) a A h_l^\theta H_{1t} \sigma R^\alpha}{n}$$

By substituting in for $h_l = [\beta\theta A(1-z)(1-q_l)]^{\frac{\theta}{1-\theta}}$

$$U^c_{ict} = H_{i1t}\left[1 + \beta A(1-z)\left[\beta\theta A(1-z)(1-q_l)\right]^{\frac{\theta}{1-\theta}} \left((1-\theta)(1-q_l) + \frac{\sigma R^\alpha}{n}\right)\right]$$

By setting $\theta = \frac{1}{2}$,

$$U^c_{ict} = H_{i1t}\left[1 + \frac{(\beta A(1-z))^2 (1-q_l)}{2}\left(\frac{(1-q_l)}{2} + \frac{\sigma R^\alpha}{n}\right)\right]$$

By substituting in for $q_l = \frac{a\sigma R^\alpha \gamma(n-1)}{n^2}$

$$U^c_{ict} = H_{i1t}\left[1 + \left(\frac{\beta A(1-z)}{2n^2}\right)^2 \left(n^2 - a\sigma R^\alpha \gamma(n-1)\right)\left(n^2 + a\sigma R^\alpha(2n - \gamma(n-1))\right)\right]$$

First and second order conditions imply that U^c_{ict} is a concave function of R for $R \in (0, \widetilde{R})$.

Consider now for $\forall R > \widetilde{R}$,

$$U^c_{ict} = H_{i1t}\left[1 + \left(\frac{\beta A(1-z)}{2n^2}\right)^2 \left(n^2 - a\sigma R^\alpha \gamma(n-1)\right)^2\right] + \frac{\beta(1-z)\underline{a}\sigma R^\alpha}{n}$$

The first and second order conditions imply that U^c_{ict} is concave function of R for $R \in (\widetilde{R}, R^*)$. Thus U^c_{ict} is piecewise concave function of R for $R \in (0, R^*)$. Further note that since $\frac{\partial h_l^\theta}{\partial R} < 0$, $\frac{\partial U^c_{ict}(a^c_t = \underline{a})}{\partial R} > \frac{\partial U^c_{ict}(a^c_t = aAh_l^\theta H_{1t})}{\partial R}$ for any $R > \widetilde{R}$. In the following, we plot utility of being corrupt as a function of the natural resource endowments.

Consider now the utility of being honest when all others are corrupt

$$U^c_{int} = H_{i1t}\left[1 + \beta A h_h^\theta (1-\theta)\right] + \frac{\beta a^n_t \sigma R^\alpha (n-1)}{Nn}$$

where $a^n_t = aAH_{1t}\left(\frac{h_h^\theta + (n-1)h_l^\theta}{n}\right)$.

$$U^c_{int} = H_{i1t}\left[1 + \beta A h_h^\theta (1-\theta)\right] + \frac{\beta a A H_{1t} h_h^\theta \sigma R^\alpha (n-1)(1 + (n-1)(1-z)(1-q_l))}{Nn^2}$$

By substituting in for $q_l = \frac{a\sigma R^\alpha \gamma(n-1)}{n^2}$

$$U^c_{int} = H_{i1t}\left[1 + \beta A h_h^\theta \left((1-\theta) + \frac{a\sigma R^\alpha (n-1)\left(1 + (n-1)(1-z)\left(1 - \frac{a\sigma R^\alpha \gamma(n-1)}{n^2}\right)\right)}{Nn^2}\right)\right]$$

First and second order conditions imply that U^c_{int} is concave function of R for $R \in (0, R^*)$.

Given the behavior of both U_{ict}^c and U_{int}^c functions, we can now look at the existence of equilibrium. Consider first $R = 0$; at $R = 0$, the utility of being corrupt is $U_{ict}^c = H_{i1t}\left[1 + \beta A\widehat{h}^\theta(1-z)(1-\theta)\right]$ and the utility of being honest when every one else is corrupt is $U_{int}^c = H_{i1t}\left[1 + \beta A h_h^\theta(1-\theta)\right]$. So at $R = 0$, by comparing both utilities, we have $U_{ict}^c < U_{int}^c$. Consider now $R = R^*$; we know that at $R = R^*$, we have $q_l = 1$, $h_l = 0$, $a_t^c = \underline{a}$ and $a_t^n = aAh_h^\theta H_{1t}/n$. Given this, the utility of being corrupt will be $U_{ict}^c = H_{i1t} + \frac{\beta(1-z)\underline{a}\sigma R^\alpha}{n}$ and the utility of staying honest when all others are corrupt will be $U_{int}^c = (1-h_h)H_{i1t} + \beta A h_h^\theta H_{i1t}\left(1 + \frac{a\sigma R^\alpha(n-1)}{Nn^2}\right)$. Our assumption A1 implies that $U_{ict}^c(R = R^*) > U_{int}^c(R = R^*)$.

Thus we have at $R = 0$, $U_{ict}^c < U_{int}^c$ and at $R = R^*$, $U_{ict}^c > U_{int}^c$. Given this, and since both functions are concave, we can have two possibilities; either these functions have single crossing or they cross thrice in the range $R \in (0, R^*)$. We claim that there is a single point $\underline{R}$ such that $U_{ict}^c = U_{int}^c$. The sufficient condition for a single $\underline{R}$ would be that at $R = \widetilde{R}$, $U_{ict}^c > U_{int}^c$. Remember that $\frac{\partial U_{ict}^c(R>\widetilde{R})}{\partial R} > \frac{\partial U_{ict}^c(R=\widetilde{R})}{\partial R}$.

In the following we prove that for a size of population not very small, there is always a single crossing between U_{ict}^c and U_{int}^c. In other words at $R = \widetilde{R}$, $U_{ict}^c > U_{int}^c$.

$$U_{ict}^c(R = \widetilde{R}) > U_{int}^c(R = \widetilde{R})$$

$$H_{i1t}\left[1 + \beta A h_l^\theta(1-z)(1-q_l)(1-\theta)\right] + \frac{\beta(1-z)a_t^c\sigma R^\alpha}{n} > H_{i1t}\left[1 + \beta A h_h^\theta(1-\theta)\right] + \frac{\beta a_t^n\sigma R^\alpha(n-1)}{Nn}$$

By substituting in for $a_t^c = aAh_l^\theta H_{1t}$ and $a_t^n = aAH_{1t}\left(\frac{h_h^\theta + (n-1)h_l^\theta}{n}\right)$

$$h_l^\theta(1-\theta)x + \frac{h_l^\theta(1-z)a\sigma R^\alpha}{n} > h_h^\theta(1-\theta) + \frac{h_h^\theta a\sigma R^\alpha(n-1)}{Nn^2} + \frac{h_l^\theta a\sigma R^\alpha(n-1)^2}{Nn^2}$$

where $x = (1-z)(1-q_l)$. Note that $h_h = (\beta\theta A)^{\frac{1}{1-\theta}}$ and $h_l = h_h x^{\frac{1}{1-\theta}}$,

$$h_l^\theta(1-\theta)\left(x - x^{\frac{-\theta}{1-\theta}}\right) > h_l^\theta\frac{a\sigma R^\alpha}{n}\left(\frac{x^{\frac{-\theta}{1-\theta}}(n-1)}{Nn^2} + \frac{(n-1)^2}{Nn^2} - (1-z)\right)$$

$$(1-\theta)\left(x - x^{\frac{-\theta}{1-\theta}}\right) > \frac{a\sigma R^\alpha}{n}\left(\frac{n-1}{Nn^2}\left(x^{\frac{-\theta}{1-\theta}} + n - 1\right) - (1-z)\right)$$

By setting $R = \widetilde{R} \equiv \left(\frac{n^2}{a\sigma\gamma(n-1)}\left(1 - \frac{1}{1-z}\left(\frac{\underline{h}}{h_h}\right)^{1-\theta}\right)\right)^{\frac{1}{\alpha}}$;

$$(1-\theta)\left(x - x^{\frac{-\theta}{1-\theta}}\right) > \frac{n}{\gamma(n-1)}\left(1 - \frac{1}{1-z}\left(\frac{\underline{h}}{h_h}\right)^{1-\theta}\right)\left(\frac{n-1}{Nn^2}\left(x^{\frac{-\theta}{1-\theta}} + n - 1\right) - (1-z)\right)$$

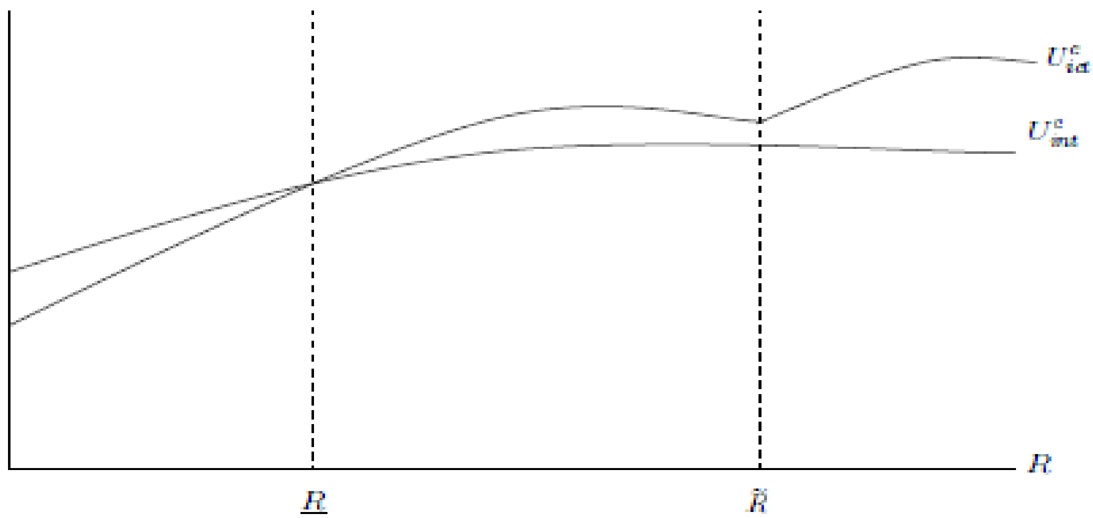
Note that $x = (1-z)(1-q_l)$ and $q_l = \frac{a\sigma R^\alpha\gamma(n-1)}{n^2}$, by substituting in for $R = \widetilde{R}$ and simplifying, we have $x = \left(\frac{\underline{h}}{h_h}\right)^{1-\theta}$. Given this, we have

$$(1-\theta)\left(x - x^{\frac{-\theta}{1-\theta}}\right) > \frac{n}{\gamma(n-1)}\left(1 - \frac{x}{1-z}\right)\left(\frac{n-1}{Nn^2}\left(x^{\frac{-\theta}{1-\theta}} + n - 1\right) - (1-z)\right)$$

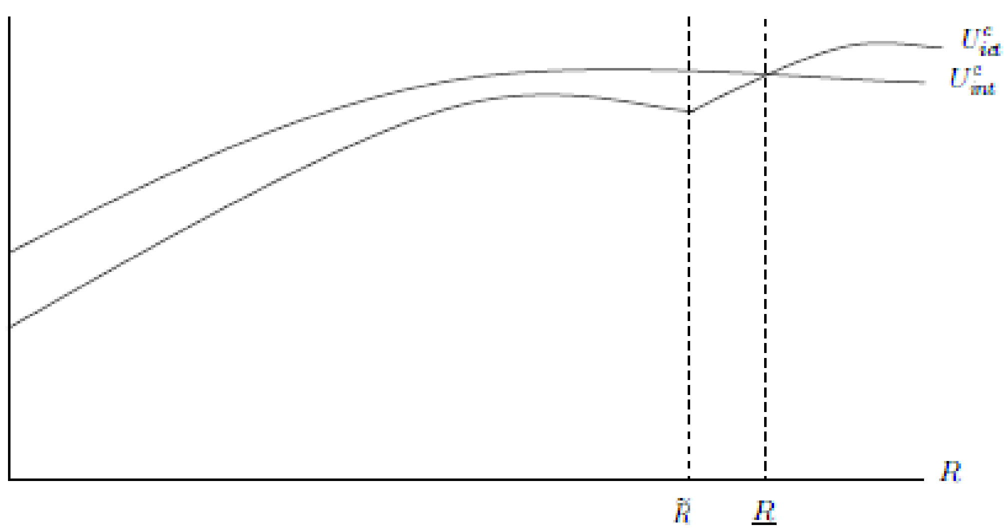
$$\gamma(n-1)(1-z)(1-\theta)\left(x - x^{\frac{-\theta}{1-\theta}}\right) > \frac{(1-z-x)}{Nn}\left((n-1)\left(x^{\frac{-\theta}{1-\theta}} + n - 1\right) - Nn(1-z)\right)$$
$$Nn(1-z)\left(\gamma(n-1)(1-\theta)\left(x - x^{\frac{-\theta}{1-\theta}}\right) + (1-z-x)\right) > (1-z-x)(n-1)\left(x^{\frac{-\theta}{1-\theta}} + n - 1\right)$$

$$N > \underline{N} \equiv \frac{(1-z-x)(n-1)\left(x^{\frac{-\theta}{1-\theta}} + n - 1\right)}{n(1-z)\left(\gamma(n-1)(1-\theta)\left(x - x^{\frac{-\theta}{1-\theta}}\right) + (1-z-x)\right)}$$

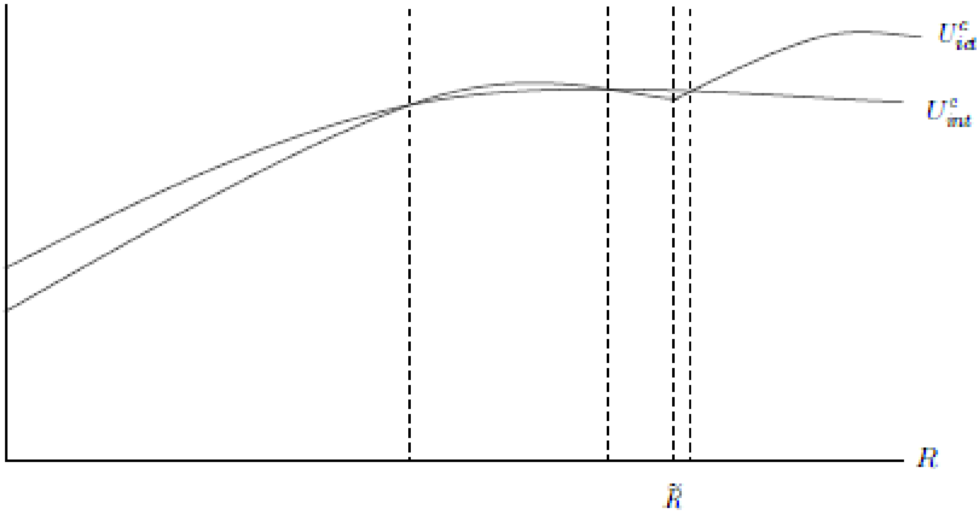
Thus,for any $N > \underline{N}$, there exists a unique $\underline{R}$ such that $U^c_{ict} = U^c_{int}$ and the low-growth equilibrium exists for any $R \geq \underline{R}$. Graphically;



It is important to note that for $N < \underline{N}$, there are two possibilities: first, there can be a unique $R > \widetilde{R}$ where both curves intersect. It is similar as if $N > \underline{N}$ with the only difference that $\underline{R}$ will be greater than $R > \widetilde{R}$.



Second, there can be three interaction points where $U^c_{ict} = U^c_{int}$.



In order to avoid multiple interaction points, we assume $N > \underline{N}$. Given our condition in the individual maximization problem which implies that the low-growth equilibrium exists for all $R < R^*$, and combining it with the above incentive constraint which implies that the low-growth equilibrium exists for all $R \geq \underline{R}$, the low-growth equilibrium exists $R^* > R \geq \underline{R}$.